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Design and Construction of Multi-Ring Apparatus for Use at High Pressures*

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A derivation is presented of equations which can be used in the design of high pressure apparatus utilizing multi-ring support. The use of these equations is illustrated by the design calculation for a multi-ring apparatus known as the anvil assembly. The particular assembly described uses three rings for external support. This assembly is capable of developing average pressures of 160 000 atm for minimum periods of two weeks. The methods and materials used in the fabrication of the assembly are also presented.

INTRODUCTION

EXPERIMENTATION in the field of extremely high pressures is generally performed by making use of pressure equipment which consists of a hydraulic press and an apparatus which will concentrate a large amount of force over a small area. This paper is concerned with the latter which, of necessity, must be designed and constructed to utilize efficiently materials with highest available strengths.

Applications with different types of high strength apparatus have been described in the literature.[1–3] Design equations for apparatus utilizing multi-ring support are derived in this paper. The derivation of these equations is performed for a class of multi-ring apparatus known as the anvil assembly. Methods applied in the derivation, however, are general. The use of these equations is illustrated with design calculations for a particular anvil assembly. This anvil assembly, when constructed in accordance with the methods outlined below, is capable of developing average pressures of 160 000 atm for a minimum period of two weeks or 100 000 atm for a period greater than two months.

NATURE OF THE ANVIL ASSEMBLY

Component parts of the anvil assembly are shown in Fig. 1. The central portions of this assembly, listed as B in Fig. 1, are called anvils, and an identical conical angle α_0 is ground on each of these anvils to give the desired strength and flat working surface. The outer portions of this assembly, listed as A in Fig. 1, represent one or more steel rings which furnish external support to the anvils. High average pressures are developed between the flat surfaces of the anvils by forcing the two portions of the assembly together with a hydraulic press. The upper limit of the average pressure developed by the anvil assembly is determined by (1) the magnitude of the flat working area of the anvils, (2) the value of the conical angle, (3) the amount of external support furnished the anvils, and (4) the method and materials of construction.

DESIGN EQUATIONS

Lamé and Clapeyron[4] were the first to obtain equations for the design of thick-walled cylinders. The expressions developed by these authors for internal pressure and

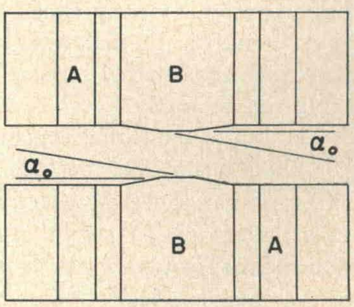

FIG. 1. Schematic diagram of the anvil assembly.

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[1] P. W. Bridgman, Proc. Am. Acad. Arts Sci. **81**, 165 (1952).
[2] R. A. Fitch, T. E. Slykhouse, and H. G. Drickamer, J. Opt. Soc. Am. **47**, 1015 (1957).
[3] H. T. Hall, Rev. Sci. Instr. **29**, 267 (1958).
[4] G. Lamé and B. P. E. Clapeyron, Memoires presentes par divers savans **4**, (1833).

radial deformation are

$$p_i=\frac{\sigma_t(b^2-a^2)+2b^2p_0}{a^2+b^2} \tag{1}$$

$$u=\left(\frac{1-\mu}{E}\right)\frac{a^2p_i-b^2p_0}{b^2-a^2}\bar{r}+\left(\frac{1+\mu}{E}\right)\frac{a^2b^2(p_i-p_0)}{(b^2-a^2)\bar{r}}, \tag{2}$$

where a=internal ring radius (in.), b=external ring radius (in.), E=modulus of elasticity (psi), p_0=external pressure on the ring (psi), $\bar{r}$=variable radius between a and b (in.), μ=Poisson's ratio (in./in.), and σ_t=maximum tangential stress at the inner radius of the cylinder (psi). These equations were obtained on the premise that materials are not stressed beyond their elastic limit.

Design calculations for an anvil assembly in which one ring furnishes the external support can be made directly by using Eqs. (1) and (2). In the usual case, the modulus of elasticity of the anvil material will be greater than that of the support ring. Thus, in using these equations an anvil under no load can be taken as completely rigid with respect to its support ring.

For an assembly with a single ring, Eq. (1) shows that the support given an anvil under no load cannot be made greater than the maximum tangential stress of the supporting steel even with a ring of infinite thickness. However, the support to the anvil can be increased above the value of the maximum tangential stress either by causing an effect analogous to autofrettaging or by use of multi-ring support.

The process of autofrettaging is performed by making the interference for assembly between the anvil and the support ring of such a value that the inner fibers are stressed beyond their elastic limit. Unfortunately, the determination of the amount that external support is increased by this process is at best a trial and error procedure. In addition, the size of the supporting ring renders almost impossible the obtaining of a uniformly hardened ring by the process of heat treating. The use of multi-ring support, on the other hand, permits the thickness of the individual rings to be kept small, and the chance of obtaining a more homogeneously hardened steel ring by heat treating is greatly enhanced. Also, the design equations derived in the following paragraphs can be used to determine the external support furnished the anvil in the multi-ring assembly.

The use of multi-ring support in design necessitates a condition for establishing the dimensions of individual rings so that the most effective use of materials is obtained. This design condition is inferred by Seely and Smith[5] for their two-ring pressure vessel in which the ratio of the outside-to-inside radius of both rings is made constant. The extension of this condition to a multi-ring assembly makes possible the determination of the radii of any ring in the assembly by

$$\begin{aligned} r_1&=r_1\\ r_2&=mr_1\\ r_3&=m^2r_1\\ r_4&=m^3r_1\\ r_5&=m^4r_1,\ \text{etc.,}\end{aligned} \tag{3}$$

where m is a constant determined by the inside radius, r_1, of the smallest ring and by the outside radius of the largest ring.

By utilizing the condition of Eq. (3), general design equations for multi-ring assemblies can be derived by using Eqs. (1) and (2). For a multi-ring assembly with dimensions satisfying Eq. (3) and containing j rings, Eq. (1) can be written for any ring as

$$p_i=\frac{\sigma_t(m^2-1)+2m^2p_0}{m^2+1}. \tag{4}$$

For the outermost ring, p_0 can be taken as zero since one atmosphere is usually small compared to the pressures developed between the ring boundaries. With zero pressure on the outside surface of the outermost ring, the internal pressure on this ring becomes

$$(p_i)_1=\frac{\sigma_t(m^2-1)}{m^2+1}, \tag{5}$$

where the subscript on p_i refers to the outermost ring. Since the inside surface of the outermost ring is in contact with the outside surface of the second outermost ring, the pressure at this position will be equal. The value of internal pressure from Eq. (5) can therefore be substituted for the value of p_0 in Eq. (4) to yield an equation for determining the internal pressure on the second outermost ring. This equation is

$$(p_i)_2=\frac{\sigma_t(m^2-1)}{m^2+1}\left[1+\frac{2m^2}{m^2+1}\right]. \tag{6}$$

The condition of equal pressure will hold for all ring boundaries in the assembly. Therefore, Eq. (6) can also be substituted into Eq. (4) for the value of p_0 so that an equation for the internal pressure on the third outermost ring is obtained. If this procedure of repeatedly substituting for p_0 in Eq. (4) is continued until ring j is reached, the resulting equation will be a geometric progression. The sum of this progression gives the internal pressure on ring j as

$$(p_i)_j=\sigma_t\left[\left(\frac{2m^2}{m^2+1}\right)^j-1\right]. \tag{7}$$

[5] F. B. Seely and J. O. Smith, *Advanced Mechanics of Materials* (John Wiley & Sons, Inc., New York, 1952).

Design calculations are facilitated when numbering of the rings in the assembly starts with the innermost ring and proceeds outward. With this type of numbering, the exponent in Eq. (7) can be adjusted in terms of the total number of rings, T, and the ring numbered f. With this adjustment, the internal pressure on ring f becomes

$$(p_i)_f=\sigma_t\left[\left(\frac{2m^2}{m^2+1}\right)^{T-f+1}-1\right] \tag{8}$$

and the external pressure on ring f becomes

$$(p_0)_f=\sigma_t\left[\left(\frac{2m^2}{m^2+1}\right)^{T-f}-1\right]. \tag{9}$$

By using Eq. (8) to calculate the external support furnished an anvil, one can determine the increase in support as more rings are used in an assembly of constant outside diameter. Results for this type of calculation are presented in Table I. Calculations for this table were made by using 150 000 psi as the value for the maximum tangential stress of the steel rings.

TABLE I. External support on a 1-in. anvil with an assembly diameter of 3 in.

Number of rings	o.d./i.d. (b/a)	External support (psi)
1	3.00	1.20×10^5
2	1.73	1.88×10^5
3	1.44	2.22×10^5
4	1.32	2.40×10^5
7	1.17	2.67×10^5
10	1.12	2.76×10^5

If the condition given by Eq. (3) is used in connection with Eq. (2), the radial deformation at a radius $\bar{r}$ for any ring in the assembly becomes

$$u=\frac{1}{E(m^2-1)}\left[\bar{r}(1-\mu)(p_i-m^2p_0)+\frac{b^2}{\bar{r}}(1+\mu)(p_i-p_0)\right]. \tag{10}$$

By using the radius a in Eq. (10), together with the internal and external pressures from Eqs. (8) and (9), the internal radial deformation of ring f is furnished by

$$(u_i)_f=\frac{\sigma_t a}{E(m^2-1)}\left\{(1-\mu)\left[\left(\frac{2m^2}{m^2+1}\right)^{T-f+1}-1+m^2-m^2\left(\frac{2m^2}{m^2+1}\right)^{T-f}\right]+(1+\mu)m^2\left[\left(\frac{2m^2}{m^2+1}\right)^{T-f+1}-\left(\frac{2m^2}{m^2+1}\right)^{T-f}\right]\right\}. \tag{11}$$

In a similar manner, the external radial deformation of ring f is

$$(u_0)_f=\frac{\sigma_t b}{E(m^2-1)}\left\{(1-\mu)\left[\left(\frac{2m^2}{m^2+1}\right)^{T-f+1}-1+m^2-m^2\left(\frac{2m^2}{m^2+1}\right)^{T-f}\right]+(1+\mu)\left[\left(\frac{2m^2}{m^2+1}\right)^{T-f+1}-\left(\frac{2m^2}{m^2+1}\right)^{T-f}\right]\right\}. \tag{12}$$

DESIGN CALCULATIONS

The usefulness of the equations derived in the last section is best illustrated by presenting the methods used in designing a particular anvil assembly. This anvil assembly, for which the design information are presented, has, for example, been tested repeatedly at average pressures of 160 000 and 100 000 atm for periods of two weeks and two months, respectively. The methods used in the fabrication of this particular assembly are given in the last section of the paper.

In this design, rings are to be specified to support a 1 in.×1 in. cylindrical anvil manufactured of cemented tungsten carbide, grade K-96, by Kennametal, Inc., of Latrobe, Pennsylvania. Grade Alco S steel manufactured by Universal Cyclops Company was selected as the material from which to construct the support rings. The modulus of elasticity of the carbide is given by the manufacturer as 94×10^6 psi, as compared to a modulus of 30×10^6 for the steel.[6] The difference in these two moduli permitted the use of the assumption, as discussed above, that anvils under no load are rigid with respect to steel rings. The Alco S steel was used with a hardness of Rockwell 50 on the "C" scale which gives a proportional limit of 167 000 psi.[6] Poisson's ratio was taken as 0.28 for the Alco S steel.

Theoretically, the initial phase in designing an anvil assembly should begin with the determination of the external support needed by the anvil under no axial loading so that radial fractures will not occur in the anvils after the maximum load has been applied. This determined external support can then be used in Eq. (8) as the value of pressure between the innermost ring and the anvil, and a combination of the three remaining variables can be selected to satisfy the equation.

Unfortunately, mathematical relationships were not available for calculating the external support required by the anvil assembly so the initial design phase could not be accomplished by the straightforward method described above. The alternate method used was to calculate the amount of external support furnished a 1-in. carbide anvil as a function of the total number of rings for various outside assembly diameters. These calculations were made by using Eq. (8) with the value of the maximum tangential stress fixed for the Alco S steel at 150 000 psi. The maxi-

[6] W. A. Boyd and S. S. Kistler, "A study of thick-walled cylinders as tensile members for high pressure vessels, with particular attention to steels and their heat treatment for maximum performance," Office of Naval Research Tech. Rept. No. III, University of Utah, Salt Lake City, Utah, 1958.

TABLE II. Values of the different radii for a 3-ring anvil assembly.

Radius	Value (in.)
r_1	0.50
r_2	0.72
r_3	1.04
r_4	1.50

TABLE IV. Interferences for a 3-ring assembly.

Boundary	Interference (in.$\times 10^3$)
Solid core-Ring I	6.96
Ring I-Ring II	4.70
Ring II-Ring III	4.90

mum tangential stress was placed at this value to assure that lateral expansion of the anvils under axial load would not cause the innermost fibers of the steel rings to exceed their proportional limit. For each outside assembly diameter, a set of results were obtained similar to those presented in Table I. These results were then compared in order to select an outside assembly diameter and the total number of rings. The selection was based on considerations of manufacturing costs and desired maximum support. An assembly containing three rings with an overall outside diameter of three inches was selected. The values of the four radii for this assembly were then calculated by using Eq. (3). These radii are tabulated in Table II. By using Eq. (11) for the inside radius and Eq. (12) for the outside radius, the amount of radial deformation for the three rings was determined to be as presented in Table III. In order to obtain maximum support from the rings without exceeding the maximum tangential stress, the rings had to be fabricated so that when the assembly was complete, all radial deformations listed in Table III would be realized. This was accomplished by allowing for the values of interference listed in Table IV.

The final phase in the anvil assembly design consisted of determining an appropriate conical angle for the anvils. This angle was fixed at four degrees for the particular anvil assembly being discussed. This choice was based on results of experiments performed to determine the time that the assembly would withstand maximum pressure without failure.

FABRICATION OF THE ANVIL ASSEMBLY

Fabrication of the anvil assembly, the design details of which are presented in the preceding section, began with the preparation of the steel support rings. The required number of disks, approximately 1 in. thick, were cut from three pieces of Alco S bar stock with appropriate diameters. By use of the lathe, rings of three sizes were formed which had oversized outside diameters and undersized inside diameters within about 0.050 of an inch of the values listed in Table II. This extra material was left on the rings so that any decarburization or warping that might occur during heat treating could be removed in the final grinding operations.

TABLE III. Radial deformations in a 3-ring assembly.

Ring number	Internal radial deformation (in.$\times 10^3$)	External radial deformation (in.$\times 10^3$)
I	3.48	2.11
II	4.46	3.28
III	5.73	2.45

The process of hardening the Alco S steel rings was started by heating to 1750°F, the temperature required for quenching. Decarburization was minimized at this high temperature by placing the rings in a closed container and surrounding them with elemental carbon. Time in the furnace at these high temperatures was reduced by allowing the furnace to reach 1400°F before the rings were inserted and then by removing the rings for oil quenching as soon as the furnace temperature rose to 1750°F. Rings were left in the container while they were moved to the oil quenching bath so that the heat loss by radiation would be decreased. After the rings had been hardened by the oil quench, the temperature of the furnace was reduced to 1000°F, and the cold rings were inserted for annealing. The container and carbon were not used for annealing since the rate of decarburization became negligible at this lower temperature. The cold rings reduced the temperature of the furnace so that an annealing time of two hours was measured from the time that the furnace regained the original 1000°F. Tests on a hardness-testing machine verified the fact that rings heat treated in accordance with the above method invariably had a Rockwell hardness of 50 on the "C" scale.

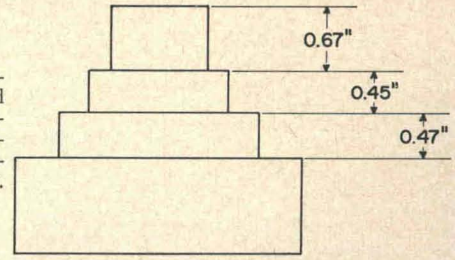
Two methods of assembly are presently specified on mechanical designs of equipment requiring interference fits. One method utilizes heating and cooling of the component parts so that the corresponding expansion and contraction will allow assembly. With the other method, interference tapers are ground on mating parts so that they can be forced together to give the desired radial deformation. In order to use the first method of assembly for the rings, the heating necessary to obtain the correct amount of expansion had to be of such a nature that the hardness of the steel was not affected. Since the required heating affected the degree of hardness, the second method of assembly was used. Therefore, after heat-treating had been completed, the flat surfaces of the rings were surface ground to give the desired height, and the inner and outer radial surfaces of each ring were ground with a taper of $\frac{1}{8}$ in./ft. The solid carbide cylinder was ground on the lathe with the same taper, a tool post grinder having a diamond wheel being used.

Values of interference specified in Table IV had to be obtained upon assembly of the component parts. Higher values of interference than those specified caused the rings to fracture when the assembly was made. With lower values, the external support on the carbide anvils was insufficient to prevent radial fracturing. In order to obtain the correct values for the interference, a gauging method was used whereby the vertical projection of each ring and anvil was measured with respect to another ring before assembly. The arrangement used to gauge the rings and anvil before assembly is shown in Fig. 2. Values for the vertical projections in Fig. 2 were calculated by using the value of the taper ground on the rings and anvils in connection with the values of interference listed in Table IV.

Owing to the fact that under an axial load the greatest increase in stress occurs at the inner radius of the inner ring of the assembly, it was found desirable to increase the value of the proportional limit of the steel at this point by the process of autofrettaging. This strengthening process was applied to the inner ring by overstressing the inner fibers on a hardened steel mandrel ground with the same taper as the rings. These inner rings were placed on the mandrel while the i.d. was still 0.010 in. under size and forced down the mandrel for a distance of one inch, a process which overstressed the inner fibers. These rings were then removed and ground to the final dimension for assembly.

Assembly of the solid core and rings was accomplished by lubricating all surfaces with molybdenum sulfide, used as an extremely fine powder suspended in a volatile solvent.

FIG. 2. Arrangement of rings and anvil for determining the values of interference to be obtained on assembly.



All rings and anvils were pressed together in a 30-ton press at one operation. Care had to be exercised during this operation as outer rings sometimes failed and were shot from the assembly. Also, outer rings were observed to fail for a period of as long as four hours after pressing. Therefore, final machining operations were not performed on rings until at least this period of time had elapsed.

The final machining operation consisted of making a fine cut with the diamond wheel on both ends of the solid carbide core to ensure parallel surfaces and then of cutting the four-degree conical angle to give the desired diameter for the flat working surface.

Anvil assemblies in which anvils fractured after long periods of operation were not discarded but were refinished. Refinishing was accomplished by first grinding the steel of the rings below that portion of the anvil which had fractured. The anvil was then resurfaced, and a new conical angle was ground. All grinding operations in the refinishing process were performed on the lathe, the tool post grinder being used. These refinishing operations were continued after fracturing until either unit of the anvil assembly was ground below a height of 0.40 in. Units below this height would not support a high total thrust and were therefore discarded.